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BPS string webs in the 6-dim (2,0) theories

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Abstract: In the Coulomb phase of the 6-dim (2,0) superconformal theories, the 1/4, 1/8, 1/16 BPS selfdual string webs are argued to exist such that the spatial SO(5) and internal SO(5) rotations are correlated. The basic constituents are 1/2 BPS strings and 1/4 BPS string junctions. One support comes from the existence of the similar BPS dyonic monostring webs in 5-dim maximally supersymmetric gauge theories. Another comes from the study of the supersymmetry of the intersecting M2 brane stripes terminating on M5 branes. We also discuss the related BPS webs in little string theories and other theories.

Keywords: M-Theory, Conformal Field Models in String Theory, Brane Dynamics in Gauge Theories, Solitons Monopoles and Instantons.

Contents

1. Introduction and conclusion

There are several mysteries in the 6-dim (2,0) superconformal theories, which were found from the study of M5 or IIA NS 5 branes world volume theory and IIB string theory on a ALE singularity [1−4] and M theory on $AdS_7 \times S^4$ [5]. While they are Lorentz invariant, the theory seems to be non-local field theory with some nonabelian two-form tensor fields of selfdual field strength and no coupling constant. In addition from the study of the near extremal black holes, these theories with the $\mathrm{U}(N)$ group seems to have N^3 degrees of freedom in large N from the black hole calculation [6], the conformal anomaly calculation using the AdS/CFT correspondence [7], and the flavor anomaly calculations [8, 9], leading to a speculation that maybe basic constituents are three-index objects instead of two-index objects from the adjoint representation.

In this article we argue that in the Coulomb phase of the (2,0) superconformal theories, there exist 1/4, 1/8, 1/16 BPS selfdual string webs which are made of basic 1/2 BPS selfdual two-indexed strings, and 1/4 BPS planar three-indexed string junctions. The spatial rotation group SO(5) is tightly correlated to the internal R-symmetry SO(5). The low energy theory of their compactification on a circle are maximally susy 5-dim Yang-Mills theories. We find the BPS equation for the dyonic monostring webs which are dimensional reduction of the selfdual string web. We also find many analogous brane webs in little string theories and other theories.

The A_N group (2,0) theories arise as the low energy theory of very closely lying $N+1$ parallel M5 branes at the limit where the gravity is decoupled. The abelian theory is described by an antisymmetric tensor field of selfdual field strength and 5 scalar fields and eight Weyl fermions [10, 11]. As the field strength is selfdual, the nonabelian theory is purely quantum mechanical, and its nature is mysterious. While there are some attempts [12, 13], we do not know the theory well enough to write down the BPS equations. In the Coulomb phase where the five scalar fields take nonzero expectation values and so the gauge symmetry is broken spontaneously to abelian subgroup, we know that there exist 1/2 BPS selfdual strings. Such BPS selfdual strings were found in the DBI action of

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a single M5 branes [14]. With further topological correction to the abelian field equations, the four coordinates of the transverse direction to a selfdual string and the four remaining scalar fields are shown to be correlated [15]. (For a more recent attempt to understand the selfdual string in SU(2) theory from the generalization of the Nahm equation, see, for example, ref. [16, 17].)

As the (2,0) theory is not directly approachable, we consider two somewhat complimentary approaches to the theory. First is to study the compactification of the (2,0) theory on the circle, whose low energy dynamics is the 5-dim maximally supersymmetric gauge theories [3]. We find the explicit BPS equations for the dyonic monostring webs and study the simplest junction in the low energy dynamics of two almost parallel distinct monostrings. Our BPS equations for the monostring webs which locks the spatial SO(4) and internal SO(4) of $SO(5)_R$ symmetry turn out to be quite similar to those appeared in the recent work on the Langlands program [18]. Also our monostring juctions may have some relevance to the operator product expansion of the 'tHooft operators, which indicates how two monopole creation operators merge to a single monopole creation operator.

Another is the study of the supersymmetry of the webs of M2 brane stripes interpolating the M5 branes by using the world volume supersymetry [19, 20], before the decoupling limit. We will show that the configuration has 1/32 supersymmetry, implying that the webs has 1/16 supersymmetry on M5 branes. A circle compactification along the M5 world volume leads to the D4 branes in type IIA string theory. The BPS webs of M2 brane stripes become the web of D2 branes and fundamental strings interpolating these parallel D4 branes in type IIA, which in turn describe the dyonic monostring webs.

A circle compactification of the space transverse to M5 branes leads to the (2,0) little string theories in the decoupling limit, We show that there exist the BPS webs of selfdual strings and little strings in the Coulomb phase of these theories. We also point out that there can be BPS webs in the (1,1) little string theories and other theories. (For a review of little string theories, see refs. [21].)

Our BPS string webs are somewhat similar to the (p,q) string junctions and webs in type IIb string theory [22, 23]. In the M-theory context, these sting webs are understood as the holomorphic embedding of M2 branes, which allows 10-dimensional webs with 1/16 supersymmetry. Our web for the little string theory can be regarded as the judicious cutting of some of these M2 brane webs by M5 branes without supersymmetry breaking. Our analysis indicates that M5 world volume are, say, the real part of the complex 5-dim space, indicating the junctions should always appear on the 5-brane world space.

Our work is also partially motivated by trying to understand the 1/4 BPS magnetic monopoles in the (2,0) theories. These dyons are represented by the(p,q) string junctions ending on D3 branes [26, 27]. Their configuration has been also studied in the M theory context with T^2 compactification of the M5 world volume [28]. However in this work, the self-dual string webs has not appeared at all.

Our consideration of the BPS webs of branes interpolating higher dimensional branes can appear in many other places. One can consider the 1/4 BPS webs of D5 branes interpolating D7 branes. The field theory on D5 branes is the N=1 4-dim supersymmetric Yang-Mills theories. These field theories may be interesting as the 5-dim field theories on

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(p,q) 5 brane webs [29].

Our investigation shows that there exist new classes of BPS webs of strings and branes in field theories and M-theories. We expect that there is a lot of degeneracy in the web configuration for a given boundary condition, whose counting may have relevance with the black hole entropy. Our work also indicates that the fields appear in (2,0) conformal theories belong to the adjoint representation of the gauge group. In the Coulomb phase the objects of three group index appear as the string junctions. We hope that our work is a step forward to understand the nonabelian nature of the (2,0) conformal theories.

The plan of this work is as follows. In section 2, we study the BPS equations and the webs of dyonic monostrings in the 5-dim maximally supersymmetric gauge theories. In section 3, the supersymmetry of the M2 brane junctions interpolating M5 branes are studied. In section 4, we make a circle compactification of the transverse space of the M5 brane and study brane webs in little string theories. We also study the webs in more general context.

2. Dyonic monostring webs

The (2,0) theories after the compactification on a circle can be described in the low energy by the 5-dim maximally supersymmetric Yang-Mills theories. This 5-dim theory is not renormalizable and is completed in the ultraviolet region by the compactified (2,0) theories. The size of the compact circle is proportional to the Yang-Mills coupling constant $e^2/4\pi$, and the (2,0) theory is the strong-coupling limit of the 5-dim theory. The Kaluza-Klein modes on the circle appear as instantons, the selfdual strings wrapping the circle appear as the W-bosons, and the straight strings appear as the magnetic monostring. They are all 1/2 BPS objects.

With the five scalar fields $\phi_A, A = 1,2,3,4,5$, the bosonic Lagrangian is

$$\mathcal{L} = -\frac{1}{2e^2}\left\{\mathrm{tr}\, F_{MN}F^{MN} + 2\mathrm{tr}\, D_M\phi_A D^M\phi_A + [\phi_A,\phi_B]^2\right\}, \tag{2.1}$$

where $D_M\phi_A = \partial_M\phi_A - i[A_M,\phi_A]$ with $M,N = 0,1,2,3,4$. The Gauss law is

$$D_aF_{a0} + i[\phi_a, D_0\phi_a] + i[\phi_5, D_0\phi_5] = 0\,. \tag{2.2}$$

We introduce the notation $a = 1,2,3,4$, which will be used to relate the SO(4) spatial symmetry to the SO(4) part of SO(5) internal space. The conserved energy is

$$\begin{aligned} H = \frac{1}{e^2}\int d^4x\, \mathrm{tr}\Big\{ & \sum_a F_{a0}^2 + \sum_{a\neq b}\frac{1}{2}F_{ab}^2 + \sum_a (D_0\phi_a)^2 + \sum_{a,b}(D_a\phi_b)^2 \\ & +(D_0\phi_5)^2 + \sum_a (D_a\phi_5)^2 - \sum_a [\phi_5,\phi_a]^2 - \frac{1}{2}\sum_{a\neq b}[\phi_a,\phi_b]^2\Big\}\,. \end{aligned} \tag{2.3}$$

By using the gauss law, we rewrite the energy as

$$
\begin{aligned}
H = \frac{1}{e^2}\int d^4x \,\mathrm{tr}\Big\{ & \sum_a (F_{a0} - D_a\phi_5)^2 + (D_0\phi_5)^2 + \sum_a (D_0\phi_a + i[\phi_5,\phi_a])^2 \\
& + \frac{1}{2}\sum_{a\neq b}\Big(F_{ab} - \epsilon_{abcd}D_c\phi_d + i[\phi_a,\phi_b]\Big)^2 + \Big(\sum_a D_a\phi_a\Big)^2\Big\} \\
+ Q_M + Q_E\,, &
\end{aligned} \tag{2.4}
$$

where the topological magnetic and electric energies are, respectively,

$$
\begin{aligned}
Q_M &= \frac{1}{e^2}\int d^4x\, \partial_a \,\mathrm{tr}\, \Big\{\epsilon_{abcd}\big(\phi_b F_{cd} + \frac{i}{3}\phi_b[\phi_c,\phi_d]\big) + (\phi_b D_b \phi_a - \phi_a D_b\phi_b)\Big\}, \\
Q_E &= \frac{1}{e^2}\int d^4x\, \partial_a\, \mathrm{tr}(F_{a0}\phi_5)\,.
\end{aligned} \tag{2.5}
$$

The energy bound is saturated by the BPS configurations which are static in time by a gauge choice and satisfy $A_0 = \phi_5$ and

$$
\begin{aligned}
F_{ab} - \epsilon_{abcd}D_c\phi_d + i[\phi_a,\phi_b] &= 0\,, \\
D_a\phi_a &= 0\,, \\
D_a^2\phi_5 - [\phi_a,[\phi_a,\phi_5]] &= 0\,.
\end{aligned} \tag{2.6}
$$

The last equation originates from the Gauss law, and determines the ϕ_5 in the given A_a, ϕ_a solution of the first part of the BPS equations. The first seven equations have also appeared recently as eq. (3.29) of ref. [18] which discusses the geometric Langlands program. One can show the above equations imply that the BPS configurations satisfy the field equations. While the instanton number does not appear in the BPS energy, one can show easily that the instanton number remains topological even after one uses the BPS equations. Once the solution is found, one can make the spatial O(4) and internal O(4) transformations independently. After using the BPS equations, the magnetic and electric energies become, respectively,

$$
Q_M = \frac{1}{e^2}\int d^4x\, \partial_a \Big\{\partial_b(\delta_{ab}\mathrm{tr}\phi_c^2 - \mathrm{tr}\phi_a\phi_b) - \frac{2i}{3}\epsilon_{abcd}\mathrm{tr}\phi_b[\phi_c,\phi_d]\Big\}\,, \tag{2.7}
$$

$$
Q_E = \frac{1}{e^2}\int d^4x\, \partial_a^2\, \mathrm{tr}\phi_5^2\,. \tag{2.8}
$$

To consider the supersymmetry, we use the 10-dim real gamma matrices, and choose our spinors to be Majorana and Weyl. Under the supersymmetry, the gaugino field transform $\delta\lambda = \frac{1}{2}F_{PQ}\Gamma^{PQ}$, with $P, Q = 0, 1, \ldots 9$ becomes

$$
\begin{aligned}
\delta\lambda = & \frac{1}{2}\Gamma^{ab}\Big(F_{ab} + i[\phi_a,\phi_b]\Gamma^{ab,4+a,4+b} - \epsilon_{abcd}D_c\phi_d\Gamma^{abc,4+d}\Big)\epsilon \\
& +\Gamma^{15}\Big(D_1\phi_1 + D_2\phi_2\Gamma^{1256} + D_3\phi_3\Gamma^{1357} + D_4\phi_4\Gamma^{1458}\Big)\epsilon \\
& +\Gamma^{a0}(F_{a0} - D_a\phi_5\Gamma^{09}) + \Gamma^{0,4+a}(D_0\phi_a + i[\phi_5,\phi_a]\Gamma^{09})\epsilon + D_0\phi_5\Gamma^{09}\epsilon\,.
\end{aligned} \tag{2.9}
$$

On the supersymmetric parameter ϵ, we can impost the mutually consistent and independent four conditions

$$\Gamma^{2345}\epsilon = -\epsilon,\ \Gamma^{1346}\epsilon = \epsilon,\ \Gamma^{1247}\epsilon = -\epsilon,\ \Gamma^{1238}\epsilon = \epsilon\,. \tag{2.10}$$

Then one can show easily that $\delta\lambda = 0$ if the self-dual equations (2.6) are satisfied. This shows that our configuration is 1/16 BPS. The spatial 1/16 BPS equation in 8+1 Yang-Mills has been obtained in ref. [30], which can be dimensionally reduced to be the first part of the BPS equations (2.6). If we have chosen different sign requirement for the ϵ, we still get the BPS equations, but the relation between the spatial rotation group SO(4) and the internal SO(4) group would not manifest.

Recall that this 5-dim theory with SU(N) gauge group arises from the low energy dynamics of parallel D4 branes very close to each other in the type IIa string theory. At the ground state $[\phi_A, \phi_B] = 0$ and so their expectation values upon multiplication of ℓ_s^2 denote the position of D4 branes in the transverse 5-dim space. Assuming the D4 world volume to be on the 01234 space of the target space, the higgs fields expectation value ϕ_a denote the position of D4 branes on the transverse x^{a+4} space. Our BPS magnetic monostring webs are made of webs of D2 brane stripes interpolating D4 branes.

The simplest case arises with only one scalar field, say ϕ_1, has nonzero expectation value, and so D4 branes lie along a line. We ignore the electric charge part for a while. The BPS equations become

$$F_{23} + D_4\phi_1 = 0,\ F_{34} + D_2\phi_1 = 0,\ F_{42} + D_3\phi_1 = 0,\ D_1\phi_1 = 0\,, \tag{2.11}$$

which are the BPS equation for magnetic monostrings along x^1 direction. For the SU(2) gauge group with the scalar field expectation value $\phi = \mathrm{diag}(v, -v)/2$, the solution for unit magnetic charge would be simply the BPS magnetic monopole solution whose radial variable is given by the radial variable transverse to the monostring. The string tension of a monostring of unit magnetic flux would be $4\pi v/e^2$.

The next simple case involves ϕ_1, ϕ_2 only, and so the D4 branes lie on a plane. The BPS equations (2.6) become

$$\begin{aligned} F_{12} + i[\phi_1, \phi_2] &= 0, & F_{34} - D_1\phi_2 + D_2\phi_1 &= 0, \\ F_{13} - D_4\phi_2 &= 0, & F_{14} + D_3\phi_2 &= 0, \\ F_{23} + D_4\phi_1 &= 0, & F_{24} - D_3\phi_1 &= 0, \\ D_1\phi_1 + D_2\phi_2 &= 0. \end{aligned} \tag{2.12}$$

With the SU(2) gauge group with the Higgs expectation value

$$\phi_1 = \mathrm{diag}(v, -v)\cos\theta/2\,,\quad \phi_2 = \mathrm{diag}(v, -v)\sin\theta/2\,, \tag{2.13}$$

the magnetic monostring solution of the above equation (2.12) is now along the line which is rotated on the 12 plane by the angle θ from the x^1 axis. In the SU(2) gauge theory, we can consider all scalar fields ϕ_a taking expectation value. The solution of our BPS equations (2.6) for the SU(2) case implies that the direction of BPS monostrings in 1234 space is identical to the line connecting two points in 5678 space given by the ϕ_a expectation value. In terms of D2 branes connecting two D4 branes, the direction of D2 branes on the D4 world volume is identical to the direction of D2 branes on the transverse space of D4 branes. This locking of the internal rotational group in the world volume and the external rotation in the transverse space would be again manifest from the study of the supersymmetry of M2 brane stripes interpolating parallel M5 branes.

With the SU(3) gauge group, the Higgs expectation value

$$\phi_1 = \mathrm{diag}(\mu_1, 0, -\mu_2), \quad \phi_2 = \mathrm{diag}(0, c, 0)/2 \tag{2.14}$$

denote the position of the three D4 branes on the transverse x^5, x^6 plane upon the multiplication of ℓ_s^2. There can be three magnetic monostrings, represented by the D2 branes connecting two of three D4 branes. We order three D4 branes by their positions are $(\mu_1, 0), (-\mu_2, 0), (0, c)$ on the 56 plane. We put μ_1, μ_2, c to be positive for the convenience. The tension of three magnetic monostrings become

$$T_{12} = \frac{4\pi}{e^2}(\mu_1 + \mu_2)h \ , \ \ T_{13} = \frac{4\pi}{e^2}\sqrt{\mu_1^2 + c^2} \ , \ \ T_{23} = \frac{4\pi}{e^2}\sqrt{\mu_2^2 + c^2} \ . \tag{2.15}$$

Each magnetic monostrings as the solution of the BPS equation (2.12) would have tilting on 12 plane determined by the Higgs expectation value. Thus $\tan\theta_{12} = 0, \tan\theta_{13} = -c/\mu_1$, and $\tan\theta_{23} = c/\mu_2$. We move these three monostrings keeping their directions and so they meet at a point in 12 plane. Then we make a junction of three monostrings. One can see easily that the tension balance condition, which are $T_{12}\cos\theta_{12} = T_{13}\cos_{13} + T_{23}\cos_{23}$ and $T_{12}\sin\theta_{12} = T_{13}\sin\theta_{13} + T_{23}\sin\theta_{23}$, are satisfied.

One could ask how BPS three D2 brane stripes are joining together and form a junction. Note that any two of three D2 brane stripes are rotated from each other in 4-dim space (2 on D4 world volume and 2 on D4 transverse directions), and so any two of them are meeting at a point in 4-dim space as any nonparallel 2-dim planes in 4-dim meets generically at a point. We expect that three D2 stripes when continued without terminating meet pairwise at three different points. When one moves each of these stripes parallel to oneself and put these three points close to each other, one may be able to cut and remove a half of each stripes and make a junction. Obviously, it is a difficult procedure. One thing clear is that near the junction, the configuration becomes very nonabelian and so the position where D2 brane stripes meet becomes fuzzy. If one uses the DBI action to understand the junction, we think that the nonabelian version leads to a better picture as our field theory approach indicates.

We have somewhat different tools to check this fact. Let us recall the low energy dynamics of two distinct monopoles, which is described by the Taub-NUT metric [31, 32]. With two non-proportional scalar field expectation value, the nonlinear sigma model of the Taub-NUT metric would be modified by the nonlinear sigma model with potential for the moduli coordinates [33, 34]. One expects naively that the same Lagrangian goes over to the low energy effective Lagrangian for two parallel magnetic monostrings, where the typical wave length for the fluctuations is much larger than the separation between strings. Thus we consider the above SU(3) model with the parameter c of ϕ_2 expectation value being very small.

Thus we consider two parallel monostrings of tension T_{13} and T_{23}, which are combined to a single string of tension T_{12}. Suppose they are lying along the $x^1 = x$ axis and their relative position $\mathbf{r} = (x^2, x^3, x^4)$ is given along the transverse direction. As the relative position and phase are now the functions of time and world sheet coordinate $x = x^1$, the relative moduli space dynamics is described by 1+ 1 dimensional nonlinear sigma model,

$$\mathcal{L}_{1+1} = -\frac{g\mu}{2}\left((1+\frac{1}{2\mu|\mathbf{r}|})(\partial_\alpha \mathbf{r})^2 + \frac{1}{4\mu^2(1+\frac{1}{2\mu|\mathbf{r}|})}(\partial_\alpha\psi + \mathbf{w}\cdot\partial_\alpha\mathbf{r})^2\right) - \frac{g}{2\mu}\frac{c^2}{1+\frac{1}{1+2\mu|\mathbf{r}|}}\,, \tag{2.16}$$

where the relative tension $\mu = \mu_1\mu_2/(\mu_1+\mu_2)$ and $g = 4\pi/e^2$. We are considering the limit where c is much smaller than μ_1, μ_2.

We expect that the composite monopole string of tension $T_{12} = g(\mu_1 + \mu_2)$ lying along x^1 direction gets split to two monopole strings of tension T_{13} and T_{13} on x^1, x^2 plane. To find the configuration, let us assume that two strings are separated along the $\mathbf{r}(x = x^1) = (y = x^2, 0, 0)(x)$. The relevant energy becomes

$$E_{\rm string} = \int dx \left\{\frac{g\mu}{2}\left(1+\frac{1}{2\mu|y|}\right)(\partial_x y)^2 + \frac{g}{2\mu}\frac{c^2}{1+\frac{1}{2\mu|y|}}\right\}. \tag{2.17}$$

We can reexpress it as

$$E_{\rm string} = \int dx\ \frac{g\mu}{2}\left(1+\frac{1}{2\mu|y|}\right)\left(\partial_x y \mp \frac{c}{\mu(1+\frac{1}{2\mu|y|})}\right)^2 \pm gc\int dx\partial_x y\,. \tag{2.18}$$

The solution $y(x)$ for the BPS equation is implicitly given by

$$2\mu y e^{2\mu y} = e^{2cx}\,, \tag{2.19}$$

for the junction point at $x = y = 0$. The separation parameter $y(x)$ goes to zero exponentially fast when $x \to -\infty$ and grows linearly with x with slope c/μ for large x. For two string branches emerging from the $1-3$ string would be described by the positions $y_1(x)$ and $y_2(x)$. The center of mass position remains zero and so $\mu_1 y_1 + \mu_2 y_2 = 0$. As the relative position $y = y_1 - y_2$, we see that two string positions diverge like $y_1(x) = cx/\mu_1$ and $y_2(x) = cx/\mu_2$. Not only the angles of these two strings are exactly what expected from the tension valence, the boundary term (2.18) for the energy matches the total sum of energies which takes the slanting of the strings into account. Thus the moduli space dynamics of two distinct monostrings gives the junction configuration.

3. Webs of M2 brane stripes terminating at M5 branes

To argue that there exist 1/4, 1/8, 1/16 BPS webs of selfdual strings in the (2,0) conformal theories, we return to the M theory origin of these theories. For simplicity, we consider the SU(N) gauge group arises from the N M5 branes. The webs of selfdual strings would

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appear as the webs of M2 brane stripes interpolating parallel M5 branes in the M theory. The consideration of the preserved supersymmetry of the web configuration is more or less similar to the monostring web case.

Let us start with the SU(2) case with the two parallel M5 branes with the world voluem on the 012345 space and take arbitrary positions on $6789\bar{1}\bar{0}$ space. They are 1/2 BPS configuration of the M theory as the spinor parameter ϵ for the kappa symmetry of these M5 branes satisfies in the 11-dim gamma matrices

$$\Gamma^{012345}\epsilon = \epsilon \,. \tag{3.1}$$

It is possible for M2 branes to end on M5 branes. They are BPS as long as M2 branes and M5 branes are planar and M2 branes are orthogonal to M5 branes [35]. The preserved supersymmetry is identical to that for the intersecting M5 and M2 branes without termination.

Let us say first two M5 branes (I,II) are separated along the x^{10} coordinate only. We now consider to put M2 brane stripes connecting these M5 branes, where all M2 branes are lying on $5\bar{1}\bar{0}$ plane. The spinor for the kappa-symmetry on the M2 brane (I,II) connecting these two M5 branes would satisfy

$$\Gamma^{05\bar{1}\bar{0}}\epsilon = \epsilon \,. \tag{3.2}$$

The above condition is compatible with the condition (3.1) on M5 and so the configuration is 1/4 BPS or has 8 supersymmetries. Many such parallel M2 brane stripes can be inserted maybe with different 1234 space. It describes the parallel selfdual strings in the (2,0) conformal theories.

Let us now imagine adding one more M5 brane (III) whose location is a bit deviated from the x^{10} axis to the x^9 axis. Now we have three M5 branes in the $9\bar{1}\bar{0}$ plane, and so three possible M2 branes connecting them. This is the M theory version of the monostring junction considered in the previous section. The coordinates of three M5 branes (I,II,III) on (x^9, x^{10}) plane to be $(0,0), (0,1), (-sin\alpha, cos\alpha)$. For two M5 branes, say (I,III), we start with angle $\alpha = 0$ and an M2 brane lying on the x^5, x^{10} plane, interpolating (I,III) M5 branes and then increase α and rotate the M2 brane stripe simulaneously in both the $4,5$ plane and the $9\bar{1}\bar{0}$ plane by the same angle. Thus the M2 brane stripe has rotated as M5 brans on x^9, x^{10} planes together, so that it remains interpolating (I,III) M5 branes. In addition its world line on x^4, x^5 plane has rotated. The spinor parameter for the κ symmetry of the second M2 brane (I,III) is preserved if

$$\Gamma^{09\bar{1}\bar{0}} e^{\alpha(\Gamma^{45} + \Gamma^{9\bar{1}\bar{0}})}\epsilon = \epsilon \,, \tag{3.3}$$

which is true for any angle α if the condition (3.2) and the following equation holds:

$$\Gamma^{049}\epsilon = \epsilon \,. \tag{3.4}$$

One can now do the similar procedure for the M2 brane stripe interpolating the (II,III) M5 branes. It would remain BPS with the same above supersymmetry condition. Thus we can now have a BPS configuraiton of three M2 brane stripes connecting three parallel M5 branes lying on x^9, x^{10} plane. Three M2 branes are intersecting each other in angles [36, 37], remaining BPS. If they are not terminating on M5 branes, two intersecting M2 branes are meeting at a point in $4591\bar{0}$ space. But as they are terminating on M5 branes and so have effective thickness [38], and so their intersection region becomes nonabelian and fuzzy. When these three M2 branes terminating on M5 branes are meeting at the same point or fuzzy region, they can make a BPS planar junction instead of continuing as the straight stripes. While it is a very nontrivial configuration, the tension balance and the five dimensional monopole string junction support this possibility.

This identification of the two rotation groups $\mathrm{SO}(5)_S \times \mathrm{SO}(5)_T$ of M2 brane stripes in the 12345 space and in the transverse $678910̄$ is essential for the supersymmetry to be maintained. Thus one can add more M5 branes with arbitrary location in $67891\bar{0}$ space and M2 branes stripes in similar manner and the total configuration will remain supersymmetric if the following independent and mutually consistent conditions are satisfie,

$$\Gamma^{012345}\epsilon = \epsilon\,, \quad \Gamma^{027}\epsilon = \epsilon\,, \quad \Gamma^{038}\epsilon = \epsilon\,, \quad \Gamma^{049}\epsilon = \epsilon\,, \quad \Gamma^{011\bar{0}}\epsilon = \epsilon\,. \tag{3.5}$$

From the condition that $\Gamma^{012\cdots 91\bar{0}} = 1$ for 11-dim gamma matrices, the imposing conditions (3.5) on M2 branes implies that $\Gamma^{016}\epsilon = \epsilon$. Thus we would have the 1/32 BPS web configurations of M2 brane stripe connecting parallel M5 branes.

4. Little string theories and other theories

One can imagine compactifying some spatial direction of the above M5-M2 brane configuration. Let us first start with compactifying $x^{1\bar{0}}$, which does not interfere with the above supersymmetry argument. For two M5 branes lying on $91\bar{0}$ plane, one can have now two kinds of BPS M2 brane stripes connecting these M5 branes. One wrapping clockwise and another wrapping counter-clockwise such that such that the total winding number of x^{10} is unity. When two M5 branes do not line on the same x^9 coordinates, these two kinds of M2 branes stripes are attracted to each other trying to form a M2 brane which winds $x^1 0$ once. (This is similar to the case where calorons appears a composite of magnetic monopoles [39].) Thus, naturally they can form a junction. As the compactifying the x^{10} is like introducing infinitely number of $M5$ branes and so there can be more complicated junctions with more winding numbers. Adding more M5 branes with different transverse directions allow webs of less supersymmetry and non-planar as in the previous case. With more M5 branes, one can now build the BPS webs of M2 brane stripes which interpolate M5 branes and the whole M2 branes which wrap x^{10} integer times.

In the little string theory limit where the gravity decouples in the above picture, a BPS little string of $(2,0)$ theory which arises as M2 brane winding the x^{10} once, appears as a singlet under the gauge symmetry. The above junction would appear on the SU(2) (2,0) little string theory as the planar junction of a little string and two complimentary selfdual strings on 45 plane. Similary, the webs would become the BPS webs of little strings and selfdual strings in the (2,0) little string theories.

Let us now consider compactifying the world sheet volume of two M5 branes, which have two different $x^{1\bar{0}}$ positions. Let us try to compactify x^4, x^5 direction of M5 direction.

The M2 brane sheets lying along x^4, x^5 and $x^9, x^{\bar{10}}$ direction and connecting two M5 branes would appear (p,q) dyons on the 3+1 dim field theory along $0, 1, 2, 3$ direction of wrapped M5 branes. We have another BPS (-p,-q) dyons by warping another $M2$ stripes which wrap $x^{\bar{10}}$ complimentary way and connect the same two M5 branes. In general these two distinct BPS M2 stripes have different tilting angle on x^4, x^5 plane. At least for the special case of the torus shape and M5 positions, one can wind these two M2 brane stripes opposite way in 4-5 torus so that their charge cancel each other. We suspect that in generic case there would be web configurations of these stripes which would have zero charge.

Without costing the divergent energy nonzero charge, one can now further compactify an additional M5 world sheet direction, say x^3. Then if there is additional M5 branes whose location is not aligned in x^8 direction, one can have nontrivial wrap on 5-3 torus, again making the total charge zero. Similarly one can compactify x^2 direction with more M5 branes not aligned along x^7 direction. This configuration would be now 1/32 BPS as the supersymmetry (3.5) indicates. Now we have effective 1+1 world sheet theory in Coulomb phase of M5 branes wrapping T^4 of T^5 space, on which there are a lump of tangled web of M2 stipes.

Once one compactify x^1 direction similar way, all M5 branes are pulled together in the transverse direction and tends to align along the compact direction x^6. It would be interesting whether still there is 1/32 BPS configurations of lumps in R^{4+1} space with T^6 compactification, with M5 branes wrapping T^5. They would be characterized by many charges with large degeneracy, and could be represented by 1/32 BPS extremal blackholes.

In somewhat different extension of the above web configurations can be found. For the little string theory of $(1, 1)$ theory in 6-dim, the low energy field theory is 6-dim (1,1) Yang-Mills theory and the monopole sheet (as generalized from 5-dim) and instanton strings are low energy description of the membrane and little strings, respectively. Instanton strings can terminate at the monopole membrane. Monopole membranes and instanton strings can form complicated BPS webs whose BPS equation in the Yang-Mills language is similar to the monopole string webs. Only change is to uplift one scalar field, say ϕ_1 for x^5 directional position to a gauge field A_5, and so the BPS equations get modified as $D_a\phi_1 \to F_{a5}$, and $-i[\phi_1, \phi_b] \to D_5\phi_b$ for $b = 2, 3, 4, 5$. The little $(1, 1)$ string theory can be lifted to the little m theory in 7 dim [40]. Thus the above membrane string web in the (1,1) little string theory can be lifted to the little m theory. Similarly the web in the (2,0) little string theory can be studied in the 8-dim little f theory.

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